# First results of $f_D$ and $f_{D_s}$ in lattice QCD with exact chiral symmetry

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**Abstract.** The decay constants of $D$ and $D_s$ are determined in quenched lattice QCD with exact chiral symmetry. Our results are: $f_D = 235(8)(14)$ MeV, and $f_{D_s} = 266(10)(18)$ MeV [1]. The latest experimental result of $f_{D^+}$ from CLEO [3] is in good agreement with our prediction.

**Keywords:** Pseudoscalar decay constant, Lattice QCD, Chiral fermion
**PACS:** 11.15.Ha, 11.30.Rd, 12.38.Gc

The pseudoscalar-meson decay constants (e.g., $f_D$, $f_{D_s}$, $f_B$ and $f_{B_s}$) play an important role in extracting the CKM matrix elements (e.g., the leptonic decay width of $D_s^+ \to l^+ \nu_l$ is proportional to $f_{D_s}^2 |V_{cs}|^2$), which are crucial for testing the flavor sector of the standard model via the unitarity of CKM matrix. Experimentally, precise determination of $f_D$ and $f_{D_s}$ will result from the high-statistics program of CLEO-c, however, the determination of $f_B$ and $f_{B_s}$ remains beyond the reach of current experiments.

Theoretically, lattice QCD provides a solid framework to compute the masses and decay constants of pseudoscalar mesons (as well as other physical observables) nonperturbatively from the first principles of QCD. Thus reliable lattice QCD determinations of $f_B$ and $f_{B_s}$ are of fundamental importance, in view of their experimental determinations are still lacking. Obviously, the first step for lattice QCD is to check whether the lattice determinations of $f_D$ and $f_{D_s}$ will agree with those coming from the high-statistics charm program of CLEO-c. This motivated our study in Ref. [1]. It turns out that our prediction of $f_D = 235(8)(14)$ MeV (posted at hep-lat on June 26) is in good agreement with the experimental result $f_{D^+} = (223 \pm 16^{+7}_{-9})$ MeV announced by CLEO-c at Lepton-Photon Symposium on July 1 [2]. Now the latest value of $f_{D^+}$ from CLEO-c has been updated to $f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4})$ MeV [3].

Here we briefly outline our computations, and refer to Ref. [1] (and references therein) for further details. The salient feature of our approach is that all quarks (no matter heavy or light) are treated on an equal footing, fully relativistic, without any heavy quark approximation for the charm quark. First, we compute quenched quark propagators for 30 quark masses in the range $0.03 \le m_q a \le 0.80$, in the framework of optimal domain-wall fermion proposed by Chiu [4]. Then we determine the inverse lattice spacing $a^{-1} = 2.237(75)$ GeV from the pion time-correlation function, with the experimental input of pion decay constant $f_\pi = 131$ MeV. The strange quark bare mass $m_s a = 0.08$ and the charm quark bare mass $m_c a = 0.80$ are fixed such that the corresponding masses extracted from the vector meson correlation function agree with $\phi(1020)$ and $J/\psi(3097)$

respectively. Then the masses and decay constants of any hadrons containing $c, s$, and $u(d)$ quarks[1] are predictions of QCD from the first principles, with the understanding that chiral extrapolation to physical $m_{u,d} \simeq m_s/25$ (or equivalently $m_\pi = 135$ MeV) is required for any observables containing $u(d)$ quarks.

We measure the time-correlation function of pseudoscalar meson ($\bar{q}Q$)

$$C_P(t) = \left\langle \sum_{\vec{x}} \mathrm{tr}\{\gamma_5 (D_c + m_Q)^{-1}_{x,0} \gamma_5 (D_c + m_q)^{-1}_{0,x}\} \right\rangle_U \tag{1}$$

(where the subscript $U$ denotes averaging over gauge configurations) for the following three categories: (i) Symmetric masses $m_Q = m_q$, (ii) Asymmetric masses with fixed $m_Q = m_s = 0.08a^{-1}$, (iii) Asymmetric masses with fixed $m_Q = m_c = 0.80a^{-1}$, where $m_q$ is varied for 30 masses in the range $0.03 \le m_q a \le 0.80$. Note that for mesons composed of strange and/or charm quarks, their masses and decay constants can be measured directly without chiral extrapolation.

The decay constant $f_P$ for a charged pseudoscalar meson $P$ is defined by

$$\langle 0 | A_\mu(0) | P(\vec{q}) \rangle = f_P q_\mu$$

where $A_\mu = \bar{q}\gamma_\mu\gamma_5 Q$ is the axial-vector part of the charged weak current after a CKM matrix element $V_{qq'}$ has been removed. Using the formula $\partial_\mu A_\mu = (m_q + m_Q)\bar{q}\gamma_5 Q$, one obtains

$$f_P = (m_q + m_Q) \frac{|\langle 0 | \bar{q}\gamma_5 Q | P(\vec{0}) \rangle|}{m_P^2} \tag{2}$$

where the pseudoscalar mass $m_P a$ and the decay amplitude $z \equiv |\langle 0 | \bar{q}\gamma_5 Q | P(\vec{0}) \rangle|$ can be obtained by fitting the pseudoscalar time-correlation function $C_P(t)$ to the usual formula

$$\frac{z^2}{2m_P a}[e^{-m_P a t} + e^{-m_P a (T-t)}] \tag{3}$$

In Fig. 1a, $m_D a$ is plotted versus $m_\pi a$, for 15 quark masses in the range $0.03 \le m_q a \le 0.10$. The data of $m_D a$ can be fitted by $m_D a = 0.816(0) + 0.101(3)(m_\pi a) + 0.298(6)(m_\pi a)^2$. At $m_\pi = 135$ MeV, it gives $m_D = 1842(15)$ MeV, in good agreement with the mass of $D$ meson (1865 MeV). In Fig. 1b, the decay constant $f_D a$ is plotted versus bare quark mass $m_q a$. The data is well fitted by the straight line $f_D a = 0.105(1) + 0.172(1) \times (m_q a)$. At $m_q a = 0$, it gives $f_D = 235(8)$ MeV, which serves as a prediction of lattice QCD with exact chiral symmetry.

The pseudoscalar meson of $\bar{s}c$ or $s\bar{c}$ corresponds to $m_Q a = m_c a = 0.80$ and $m_q a = m_s a = 0.08$. Its mass and decay constant are extracted directly from the time-correlation function, which are plotted as the eleventh data point (counting from the smallest one) in Fig. 1. The results are $m_{D_s} a = 0.878(2)$ and $f_{D_s} a = 0.119(2)$. The mass gives

[1] In this paper, we work in the isospin limit $m_u = m_d$.

$m_{D_s} = 1964(5)$ MeV, in good agreement with the mass of $D_s(1968)$. The decay constant gives $f_{D_s} = 266(10)$ MeV, which agrees with the value $f_{D_s^+} = 267 \pm 33$ MeV complied by PDG [5].

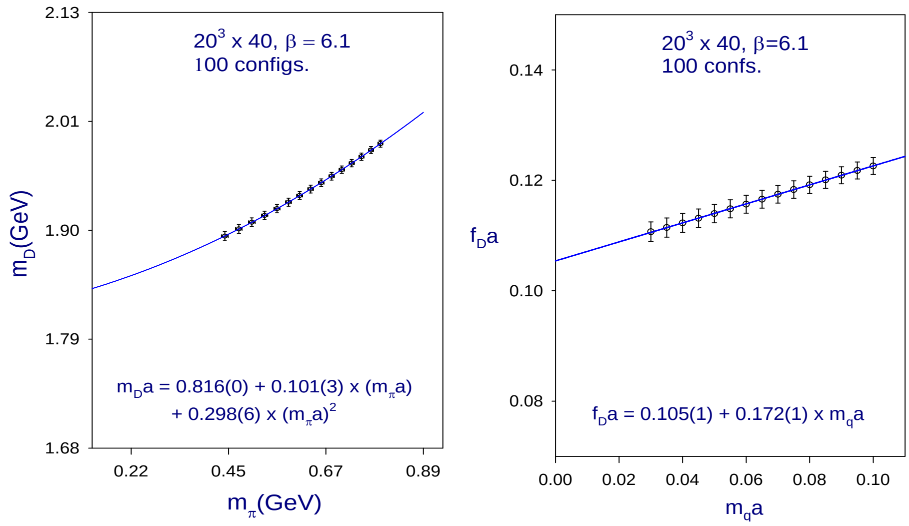


**FIGURE 1.** (a) The $D$-meson mass $m_D$ versus the pion mass $m_\pi$ for 15 bare quark masses in the range $0.03 \le m_q a \le 0.10$. The solid line is the quadratic fit. (b) The D-meson decay constant $f_D a$ versus the bare quark mass $m_q a$. The solid line is the linear fit.

To summarize, we have determined the masses and decay constants of $D$ and $D_s$, in quenched lattice QCD with exact chiral symmetry. Our results are: $m_D = 1842(15)(21)$ MeV, $m_{D_s} = 1964(5)(10)$ MeV, $f_D = 235(8)(14)$ MeV, $f_{D_s} = 266(10)(18)$ MeV, where in each case, the first error is statistical, and the second is our crude estimate of combined systematic uncertainty. The experimental value of $f_{D^+}$ turns out to be $f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4})$ MeV [3], in good agreement with our prediction of $f_D$. Now it remains to see whether the value of $f_{D_s}$ coming from the high-statistics charm program of CLEO-c would agree with our prediction determined by lattice QCD with exact chiral symmetry.

This work was supported in part by the National Science Council (ROC), and National Center for High Performance Computation at Hsinchu.

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